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Dynamic linear system identification via iterative back propagation of errors in input-output data

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Abstract

A common kind of model used for system modelling in engineering is the errors-in-variables (EIV) model, which has both noisy output and noisy input data. However, input disturbances make identification for the EIV model particularly challenging. This work addresses the challenge of identifying real-time EIV models adaptively. A recent study highlighted certain derivation mistakes in an accuracy investigation of the widely utilized Frisch technique for EIV detection. In this article, a novel approach is given to address the same modelling issue. System characteristics and the noise attributes are estimated from both the time domain and the frequency domain perspectives, with a Moving Average (MA) procedure standing in for the combined influence of the mutually independent input and output noises. To enhance the speed and accuracy of calculations and to make them possible to do online, a recursive version of the first step calculation is built. The flexibility of the suggested method to handle a variety of input processing circumstances is another bonus. The effectiveness and reliability of the proposed approach are shown by numerical simulations.

Introduction

Modelling is a crucial problem in engineering. Stochastic models are often used to represent these systems, with the input signals being treated as idealized ideals and the perturbed noises being added to the output signals. We refer to these representations as "errors-in-equation models." However, there are always external signals that alter the input of the systems; some of these signals cannot be accounted for in the output noises. Since the real physical rules of the process are more important than the prediction of future behaviour [1], it is equally important to investigate the modelling issue for such systems with noisy input output data. This kind of model, in which both the input and output measurements are subject to noise, is known as an "errors-in-variables (EIV) model [2]." Over the last several decades, researchers have paid a lot of effort to pinpointing EIV models. The usage of EIV models has spread to many fields, including econometrics, computer vision, biomedicine, chemical and image reconstruction, spectrum estimation, voice analysis, noise cancellation, and digital communications [3-8]. It is far more challenging to identify EIV models since the noise in the input measurements cannot be similar to the output error. EIV dynamic model identifiability was studied in [9, 10]. In [9], it is emphasized that second-order features cannot be used for unambiguous identification of EIV dynamic models. Therefore, identifying them requires previous information of a certain kind. In order to create estimate algorithms, identifiability must first be demonstrated [10]. Since the EIV models use noisy input data, the traditional least squares approach for errors-in-equation models no longer provides reliable estimates. A bias-compensated least squares (BCLSs) concept was presented to address this issue in [4]. The BCLS idea has been the foundation for a number of other algorithms, including those based on the Frisch scheme [7], the KL algorithm [8], the ECLS [9], the BELS [10], and others [11–15]. Although several techniques exist for determining distinct EIV models, it has always been challenging to achieve algorithmic convergence. Only a small number of works [12, 15, 16] have attempted to provide a solution to this issue. The accuracy of the Frisch scheme for EIV identification was studied in [16], where it was shown that the linearization of the scheme's three main equations yielded asymptotically Gaussian distributions for both the estimated system parameters and the noise variance. This result might be seen as theoretical backing for algorithms based on the Frisch scheme. Recent expansions and practical applications [17–20] of this work confirm the significance of this analytical outcome. Parameter estimations are required for the analysis in [16], but it is unclear how to verify that they are near to the genuine values. Convergence failure in a counterexample was discovered in [21], and certain derivation flaws in [16] were also discovered and discussed. In addition, another strategy for determining the EIV model was described in [21]. However, the model addressed in [21] was a simpler one with a stronger requirement that the input and output noise processes had the same variance, which has been hindering its application to some degree in comparison to the model concerned in [16] owing to the complexity of the identification issue.

Problem Statement

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Figure 1 depicts a simple example of a dynamic EIV system. The EIV model differs from the standard errors-in-equation model in that it accounts for noise in both input and output data. A dynamic system, which may be linear or nonlinear depending on the context, connects the unobservable true input and output processes u0(t) and y0(t). Too far, the majority of research in this area has been on linear systems.

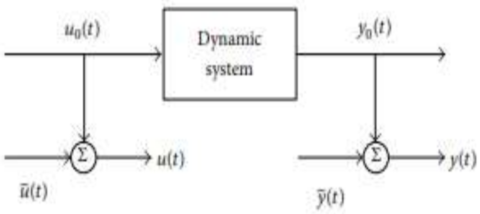

Figure 1: A basic errors-in-variables system. research in this paper.

The ARX(na, nb) model is considered here as

$$A(z)\,y_0(t) = B(z)\,u_0(t),$$

where

$$A(z) = 1 + a_1 z + \cdots + a_{n_a} z^{n_a},$$

$$B(z) = b_1 z + \cdots + b_{n_b} z^{n_b},$$

are the polynomials in the backward shift operator z. The $\{a1,\ a2, a\ ,\ b1,\ b2, bnb\ \}$are the unknown system paramenters to be identified, while the measured variables $u(t)$ and $y(t)$ are disturbed by the unknown noises $u(t)\tilde{}$ and $y(t)\tilde{}$. Thus, the input and output measurements are

$$u(t) = u_0(t) + \tilde{u}(t), \qquad y(t) = y_0(t) + \tilde{y}(t).$$

After introducing the notations

$$\theta = \left(a_1, a_2, \ldots, a_{n_a}, b_1, b_2, \ldots, b_{n_b}\right)^T,$$

$$\varphi(t)$$

$$= \left(-y(t-1), \ldots, -y(t-n_a), u(t-1), \ldots, u(t-n_b)\right)^T,$$

$$\tilde{\varphi}(t)$$

$$= \left(-\tilde{y}(t-1), \ldots, -\tilde{y}(t-n_a), \tilde{u}(t-1), \ldots, \tilde{u}(t-n_b)\right)^T,$$

the EIV system can be described as the following model

$$y(t) = \varphi(t)^T\theta + A(z)\,\tilde{y}(t) - B(z)\,\tilde{u}(t).$$

We begin with a set of assumptions to guarantee uniqueness. (A1) There is no zero of A(z) within the unit circle, hence the EIV system is asymptotically stable. There is no correlation between the noises u(t) and y(t) and the genuine input and output signals u0(t) and y0(t), as required by (A2). Both u(t) and y(t) in (A3) are white sounds with zero mean and zero correlation between them. Our goal is to use the observed regressor vector (t) to make an estimate of the system parameter vector. Since the mean and variance may both be used to characterize a noise process, it is easier to pinpoint the sources of input and output sounds that have zero mean. Therefore, it is desirable to estimate not only the system parameters but also the output and input noise variances y and u. In the following, we detail a two-step procedure that satisfies both of these components of the estimate.

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Methods of Identifying

Because of the uncertain nature of the input and output sounds, system identification for an EIV system is notoriously challenging. Since two sequences of independent random variables can be represented as an MA process which has the same spectra with the two jointly sequences [22], we will use another MA process w(t) to counteract the effect of the input noise on the EIV system described in Section 2. The system may then be transformed into an ARMAX model, with the subsequent changes required to estimate the new model's system parameters and to calculate the input/output noise variances in terms of w(t). To determine the system parameters and the noise variances y and u, a two-stage recursive estimate procedure may be developed. First, we estimated the parameters and produced the most up-to-date estimates of (t) and w(t) by using the acquired estimation of w(t 1). Second, the estimates of the noise variance y(t) and u(t) are derived from these values. The algorithm and proof will be shown below. First-stage estimate of the system's unknown parameter. Let's call the final two terms of (5) V(t) for short.

$$v(t) = A(z)\,\tilde{y}(t) - B(z)\,\tilde{u}(t),$$

where $u(t)^{\tilde{}}$ and $y(t)^{\tilde{}}$ are mutually independent with

$$E\tilde{y}(t) = E\tilde{u}(t) = 0, \qquad E\tilde{y}^2(t) = \lambda_y,$$
$$E\tilde{y}^2(t) = \lambda_u.$$

Introduce an MA(nc) process

$$w(t) = e(t) + c_1 e(t-1) + \cdots + c_{n_c} e(t-n_c),$$

where $\{e(t)\}$ is white noise with

$$Ee(t) = 0, \qquad Ee^2(t) = \lambda_e,$$
$$n_c = \max\{n_a, n_b\}.$$

It can be shown that we can find a pair of $\{ci, 0 \le i \le nc\}$ and λe such that $\{w(t)\}$ and $\{\mathrm{V}(t)\}$ have the same spectra [22], which means that $\{\mathrm{V}(t)\}$ can be represented by $\{w(t)\}$ in (8) as

$$y(t) = \varphi(t)^T\theta + w(t).$$

The

$\{c_i, 0 \le i \le n_c\}$ and λ_e

and λe are intermediate variables.

For the new model (10), denote a new parameter vector θ and a new regressor vector (t) by

$$\overline{\theta} = \left(\theta^T, c_1, c_2, \ldots, c_{n_c}\right)^T,$$

$$\overline{\varphi}(t) = \left(\varphi(t)^T, e(t-1), \ldots, e(t-n_c)\right)^T,$$

and then the EIV system (5) can be rewritten as

$$y(t) = \overline{\varphi}(t)^T\overline{\theta} + e(t).$$

In this step, we will give a recursive algorithm to identify (13). The covariance matrix of the regressor vector (t) and output variables (t) is denoted by

$$R_{\overline{\varphi}} = E\overline{\varphi}(i)\,\overline{\varphi}(i)^T,$$

$$r_{\overline{\varphi}y} = E\overline{\varphi}(i)\,y(i).$$

For convenience, introduce

$$\hat{R}_{\overline{\varphi}}(t) = \sum_{i=1}^{t}\overline{\varphi}(i)\,\overline{\varphi}(i)^T,$$

$$\hat{r}_{\overline{\varphi}y}(t) = \sum_{i=1}^{t}\overline{\varphi}(i)\,y(i).$$

Assume that the input $\{(t)\}$ is a stationary process; in the calculation, we can use the algebra means $\hat{R}(t)/t$ and $\hat{r}\varphi y(t)/t$ instead of the mathematical expectations $R\varphi$ and $r\varphi y$ in (14), as by ergodicity, we have

$$\frac{\hat{R}_{\overline{\varphi}}(t)}{t} \xrightarrow{t\to\infty} R_{\overline{\varphi}}, \qquad \frac{\hat{r}_{\overline{\varphi}y}(t)}{t} \xrightarrow{t\to\infty} r_{\overline{\varphi}y}.$$

Lemma 1 (Matrix Inversion Formula [23]). For the matrices

$A \in R^{n\times n}$, $C \in R^{n\times 1}$, and $D \in R^{n\times 1}$

the inverse matrix of

$B = A + CD^T$ is

$$\left(A + CD^T\right)^{-1} = A^{-1} - a^{-1}A^{-1}CD^TA^{-1},$$

where $a = 1 + D^TA^{-1}C$.

Simulation Examples

This section addresses some numerical evaluation of the identification algorithm presented in this paper. MATLAB 7.7 is used to do the simulations. To demonstrate its vapidness to various EIV systems, we have chosen different signal processes as the true input variables $\{u0(t)\}$ in each case: in Case A, a zero-mean Gaussian process is used; in Case B, a sawtooth signal is applied; in Case C, it is an ARMA process. The noise processes $\{(t)\}^{\sim}$ and $\{(t)\}^{\sim}$ in these cases are mutually uncorrelated white noise signals with zeromean. The robustness of the algorithm is also tested, which is shown in Case C. Case A. First, we examine how well the algorithm works for systems with Gaussian input. Consider an EIV dynamic system with $na = nb = 2$ and

$$\theta = (a_1, a_2, b_1, b_2)^T = (-0.2, -0.15, 0.3, -0.27)^T.$$

It is easy to get the system as follows, which is denoted by System 1:

$$\text{System 1: } y_0(t) - 0.2y_0(t-1) - 0.15y_0(t-2) = 0.3u_0(t-1) - 0.27u_0(t-2).$$

let the input signal $\{u0(t)\}$ be a zero-mean Gaussian process whose variance equals 1. Let the noise signals $\{(t)\}^{\sim}$ and $\{(t)\}^{\sim}$ be mutually uncorrelated white noise signals with $\lambda y = 0.2$, $\lambda u = 0.5$, which means a strong noise environment for the system. The system is simulated for $N = 8000$ steps. Calculation results are listed in Table 1, where the calculation error is defined by the standard deviation. Figures 2 and 3 show the system parameter and the noise variances estimates separately. Solid lines indicate the true values and dashed lines denote the corresponding estimates. Noting that the vertical coordinate scopes are very small in both figures, it can be seen easily that the estimates are converging fast to the true parameters

Case B. Consider another system, System 2, with sawtooth input

$$\text{System 2: } y_0(t) = \frac{z - 2z^2}{(1-0.9z)(1-0.8z)} u_0(t),$$

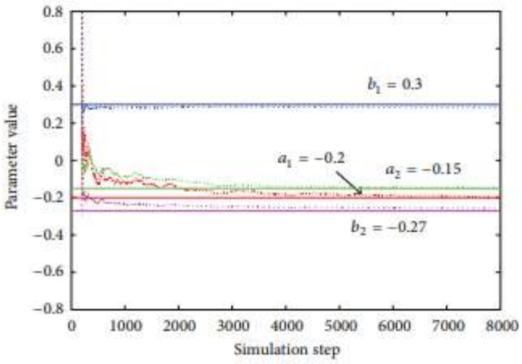

Figure 2: Estimation result of θ in System 1.

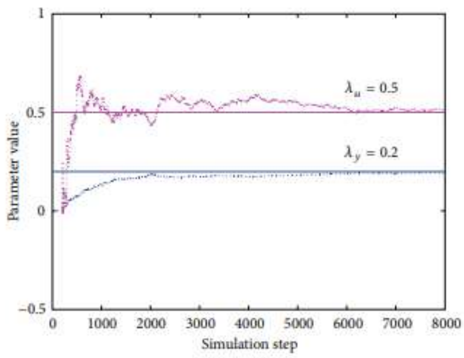

Figure 3: Estimation result of λy and λu in System 1. in which na = nb = nc = 2, and

$$\theta = (a_1, a_2, b_1, b_2)^T = (-1.7, 0.72, 1, -2)^T.$$

This is the counterexample which was presented in [21] to show the convergency of the Frisch-based method analysed in [16]. We use our proposed algorithm to identify this system under the same conditions; that is, the input sawtooth signal's amplitude equals 1 and its frequency is 10 Hz; the noises' variances are

$$\lambda_y = \lambda_u = 0.5.$$

The simulation results of the system parameters and the noise variances are all displayed in Figure 4. The true values and estimates are also denoted by solid lines, and dotted lines respectively. We can see that the algorithm has an even better performance for this kind of EIV system. All the estimates converge to their corresponding true values consummately.

Conclusions

In this study, we looked at the issue of identifying systems with dynamic errors in variables (EIVs). Cascade system modelling and camera calibration are only two examples of the many technical uses for the EIV model. The noise issue with the EIV model's input data is more problematic than with other errors-in-equation models. Recent research has highlighted various serious flaws in the earlier examination of the appealing Frisch scheme identification techniques;thus, we created an adaptive algorithm to address the same modelling issue. Due to its recursive nature, this two-step technique can estimate the system parameter vector and the noise variances with improved precision for the dynamic EIV model with mutually independent input and output noises, while also greatly reducing the computational complexity. Numerical simulations have demonstrated that the provided approach achieves high levels of accuracy and convergence quickly while maintaining strong antinomies performance. More advanced models, such as EIV nonlinear models, will be evaluated in further research, and theoretical analysis of the suggested method will be performed.

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